Deep Inelastic Scattering in Lepton-Hadron Collisions
— Probing the Parton Structure of the Nucleon with Leptons

- Basic Formalism
  (indep. of strong dynamics and parton picture)
- Experimental Development
  - Fixed target experiments
  - HERA experiments
- Parton Model and QCD
  - Parton Picture of Feynman-Bjorken
  - Asymptotic freedom, factorization and QCD
- Phenomenology
  - QCD parameters
  - Parton distribution functions
  - Other interesting topics
Lepton-hadron scattering process

\[ \ell_1(\ell_1) + N(P) \rightarrow \ell_2(\ell_2) + X(P_X) \]

Effective fermion-boson electro-weak interaction Lagrangian:

\[ \mathcal{L}_{\text{EW}}^{\text{int}} = -g_B \left[ \bar{J}_\mu^{(\ell)}(x) + J^{(h)}_\mu(x) \right] V_B^\mu(x) \]

**EW SU(2)xU(1) gauge coupling constants**

<table>
<thead>
<tr>
<th>B</th>
<th>( \gamma )</th>
<th>( W^\pm )</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_B )</td>
<td>(-e)</td>
<td>( \frac{g}{2\sqrt{2}} )</td>
<td>( \frac{g}{2 \cos \theta_W} )</td>
</tr>
</tbody>
</table>
Basic Formalism: current operators and coupling

Fermion current operator:

\[ J^{(f)}_{\mu}(x) = \overline{\psi}_f(x) \gamma_\mu (g_V - g_A \gamma^5 ) \psi_f(x) = \overline{\psi}_f(x) \gamma_\mu [g_R (1 + \gamma^5) + g_L (1 - \gamma^5)] \psi_f(x) \]

V-A couplings: or,

Left-right (chiral) couplings:

\[ \begin{array}{c|c|c|c}
\gamma & Z & W^{\pm} \\
\hline
\gamma & Q_i & T_{3L}^i - 2Q_i \sin^2 \theta_W & 1 \cdot V_{ij} \\
\hline
g_A & 0 & T_{3L}^i & 1 \cdot V_{ij} \\
\hline
\frac{g_R}{2} & -Q_i \sin^2 \theta_W & 0 & \\
\hline
\frac{g_L}{2} & T_{3L}^i - Q_i \sin^2 \theta_W & 1 \cdot V_{ij} \\
\end{array} \]
Basic Formalism: Scattering Amplitudes

Scattering Amplitudes

\[ M = J^*_\mu(P, q) \frac{g_B^2 G^\mu_\nu}{Q^2 + M_B^2} j^\nu(q, \ell) \]

Spin 1 projection tensor

\[ G^\mu_\nu = g^\mu_\nu - q^\mu q_\nu / M_B^2. \]

Lepton current amplitude (known):

\[ j^\mu(q, \ell) = \langle \ell_2 | j^\mu | \ell_1 \rangle = \bar{u}(\ell_2) \gamma^\mu [g_R(1 + \gamma^5) + g_L(1 - \gamma^5)] u(\ell_1) \]

Hadron current amplitude (unknown):

\[ J^*_\mu(P, q) = \langle P \times | J^*_\mu | P \rangle \]

Object of study:

* Parton structure of the nucleon; (short distance)
* QCD dynamics at the confinement scale (long dis.)
Cross section

\[ d\sigma = \frac{G_1 G_2}{2\Delta(s, m_{\ell_1}^2, M^2)} 4\pi Q^2 L_{\nu}^\mu W_{\mu}^\nu \, d\Gamma \]

Lepton tensor (known):

\[ L_{\nu}^\mu = \frac{1}{Q^2} \sum_{\text{spin}} \langle \ell_1 | j_{\nu}^j | \ell_2 \rangle \langle \ell_2 | j_{\mu}^j | \ell_1 \rangle \]

Hadron tensor (unknown):

\[ W_{\nu}^\mu = \frac{1}{4\pi} \sum_{\text{spin}} (2\pi)^4 \delta^4(P + q - P_X) \langle P | J_{\nu}^\mu | P_X \rangle \langle P_X | J_{\mu}^j | P \rangle \]

Object of study:

* Parton structure of the nucleon;
* QCD dynamics at the confinement scale
Expansion of $W_{\mu \nu}^\mu$ in terms of independent components

$$W_{\mu \nu}^\mu = -g_{\mu \nu} W_1 + \frac{P_{\mu} P_{\nu}}{M^2} W_2 - i \frac{\epsilon^{P_{\mu} q_{\nu}}}{2M^2} W_3 + \frac{q_{\mu} q_{\nu}}{M^2} W_4 + \frac{P_{\mu} q_{\nu} + q_{\mu} P_{\nu}}{2M^2} W_5 + \frac{P_{\mu} q_{\nu} - q_{\mu} P_{\nu}}{2M^2} W_6$$

Cross section in terms of the structure functions

$$\frac{d\sigma}{dE_2 d\cos \theta} = \frac{2E_2^2 G_1 G_2}{\pi M} \frac{1}{n_{\ell}} \left\{ g^2_{+ \ell} \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right] \pm g^2_{- \ell} \left[ \frac{E_1 + E_2}{M} W_3 \sin^2 \frac{\theta}{2} \right] \right\}$$

Charged Current (CC) processes (neutrino beams):

W-exchange (diagonal); left-handed coupling only; ....

Neutral Current (NC) processes (e,μ scat.)---low energy:

(fixed tgt): γ-exchange (diagonal); vector coupling only; ...

Neutral Current (NC) processes (e,μ scat.)---high energy

(hera): γ & Z exchanges: $G_1^2$, $G_1 G_2$, $G_2^2$ terms; ....
**Basic Formalism: Scaling structure functions**

**Kinematic variables**

\[ \nu = \frac{P \cdot q}{\sqrt{P \cdot P}} = E_1 - E_2 \]
\[ x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M \nu} \]
\[ y = \frac{P \cdot q}{P \cdot E_1 \ell} = \frac{\nu}{E_1} \]

**Scaling (dimensionless) structure functions**

\[ F_1(x, Q) = W_1 \]
\[ F_2(x, Q) = \frac{\nu}{M} W_2 \]
\[ F_3(x, Q) = \frac{\nu}{M} W_3 \]

Scaling form of cross section formula:

\[ \left( g_{\pm \ell}^2 = g_{L \ell}^2 \pm g_{R \ell}^2 \right) \]

\[ \frac{d\sigma}{dxdy} = \frac{2ME_1}{\pi} \frac{G_1 G_2}{n_\ell} \left\{ g_{+ \ell}^2 \left[ xF_1 y^2 + F_2 \left[ (1 - y) - \left( \frac{Mxy}{2E_1} \right) \right] \right] \pm g_{- \ell}^2 \left[ xF_3 y(1 - y/2) \right] \right\} \]

\( n_\ell \) is the number of polarization states of the incoming lepton.
Basic Formalism: Helicity Amplitudes

**Scattering Amplitudes**

\[ \mathcal{M}_{\lambda_1 \sigma_1}^{\lambda_2 \sigma_2} = J_{\sigma_1 m}^{* \sigma_2} (Q^2, q \cdot P) \frac{g_B^2}{Q^2 + M_B^2} d^1(\psi)^m_n j_{\lambda_1}^n (Q^2) \]

Spin 1 rotation matrix

\[ d^1(\psi)^m_n \]

Lepton current amplitude (known):

\[ j_{\lambda_1}^\lambda_n (Q) = \epsilon_\mu^n \langle \ell_2, \lambda_2 | j^\mu | \ell_1, \lambda_1 \rangle = \bar{u}_{\lambda_2} (\ell_2) \epsilon_n^{* \mu} \cdot \Gamma u_{\lambda_1} (\ell_1) \]

Hadron current amplitude (unknown):

\[ J_{\sigma_1 m}^{* \sigma_2} (Q^2, q \cdot P) = \langle P_X, \sigma_2 | J_\mu^\dagger | P, \sigma_1 \rangle \epsilon_\mu^m \]

Object of study:
* Parton structure of the nucleon;
* QCD dynamics at the confinement scale
Basic Formalism: Helicity structure functions

Forward Compton scattering amplitude for vector boson polarization $\lambda$

$$F_{\lambda} = \epsilon_{\mu}^{\lambda*}(P, q) W_{\nu}(P, q) \epsilon_{\lambda}(P, q)$$

Relations between invariant and helicity S.F.s

- $F_{\text{right}} = F_{+}(x, Q) = F_1 - \frac{1}{2}\kappa F_3$
- $F_{\text{left}} = F_{-}(x, Q) = F_1 + \frac{1}{2}\kappa F_3$
- $F_{\text{long}} = F_{0}(x, Q) = -F_1 + \frac{1}{2x}\kappa^2 F_2$

Where at high energies

$$\kappa = \sqrt{1 - \frac{Q^2}{\nu^2}} \approx 1$$

Conversely,

$$F_1 = \frac{1}{2}(F_{\text{right}} + F_{\text{left}}) = \frac{1}{2}(F_{\text{right}} + F_{\text{left}}) = F_T$$

$$F_2 = 2x(F_T + F_L) \frac{1}{\kappa^2}$$

$$F_3 = (F_{\text{right}} - F_{\text{left}}) \frac{1}{\kappa}$$

Cross section formula:

$$\frac{d\sigma}{dxdy} = \frac{yQ^2}{2\pi} \frac{G_1 G_2}{n_\ell} \left\{ g_{+\ell}^2 [F_T(1 + \cosh^2 \psi) + F_L \sinh^2 \psi] + g_{-\ell}^2 [\kappa F_3 \cosh \psi] \right\}$$

where

$$\cosh \psi = \frac{E_1 + E_2}{\sqrt{Q^2 + \nu^2}} \quad \rightarrow \quad (2 - y) \frac{M \rightarrow 0}{y}$$
Interesting and comprehensive description of the entire history of probing the structure of nuclei—from (pre-) Rutherford scattering … 1930’s …50’s (Hofstadter) …60’s, 70’s (SLAC), …SPS, Fermilab, HERA  Highly recommended!

2-lecture series by E. Tassi at CTEQ2003
The SLAC-MIT Experiment

Under the leadership of Taylor, Friedman, Kendall
(Nobel prize, 1990)
First SLAC-MIT results ~ 1969

Two unexpected results...

[Graphs showing data points and curves indicating elastic scattering and other results.]
Experimental Development
— modern experiments (high Q)

DIS

$eN$
$\mu N$
$\nu N$
$\bar{\nu} N$

SLAC
BCDMS
NMC, E665
H1, ZEUS

CDHS, CHARM
CCFR
CHORUS
NuTeV
The highest energy (anti-) neutrino DIS experiments

CCFR and NuTeV
Fermilab
The highest energy (anti-) neutrino DIS experiment

The NuTeV experiment at FNAL

v-N DIS, sign-selected beam $<E_v> \sim 120$ GeV
and continuous test beam calibration

Data taken during 1996-97

Hadronic energy: $\delta E/E \sim 0.4\%$
$\sigma E/E \sim (2.4 + 87/\sqrt{E})\%$

$\mu$ momentum
$\delta P/P \sim 1\%$
$\sigma P/P \sim 11\%$

Relative scale $(E,P) < 0.5\%$
Fixed targets results: An overview (PDG)

F₂: \(1 < Q^2 < 200 \text{ GeV}^2\)

F₃: \(1 < Q^2 < 200 \text{ GeV}^2\)

Fₐ: \(Q^2 = 2.2, 4.2, 7.5 \text{ GeV}^2\)

Fₐ: \(Q^2 = 12, 15, 20 \text{ GeV}^2\)

Fₐ: \(Q^2 = 25, 35 \text{ GeV}^2\)
The HERA Collider

The first and only ep collider in the world

Located in Hamburg

Equivalent to fixed target experiment with 50 TeV $e^\pm$

\[ \sqrt{s} = 318 \text{ GeV} \]

\[ e^\pm \quad p \]

27.5 GeV   920 GeV
Colliding beam experiments

**H1 - ZEUS**

- Uses p beam on wire target
- Goal: B - physics

**HERA-B**

- Uses e± beam on gas jet target
- Both lepton and target polarized
- Measurement of polarized structure functions
The Collider Experiments

**H1 Detector**

Complete 4π detector with:

- Tracking
  - Si-µVTX
  - Central drift chamber
- Liquid Ar calorimeter
  - $\hat{E} = E = 12\% = \sqrt{E_{[GeV]}}$ (e.m.)
  - $\hat{E} = E = 50\% = \sqrt{E_{[GeV]}}$ (had)
- Rear Pb-scintillator calorimeter
  - $\hat{E} = E = 7.5\% = \sqrt{E_{[GeV]}}$ (e.m.)
- µ chambers
- and much more…
ZEUS Detector

- Complete $4\pi$ detector with
  - Tracking
    - Si-$\mu$VTX
    - Central drift chamber
  - Uranium-Scintillator calorimeter
    - $\hat{E} = E = 18\% = E[GeV]$ (e:m:)
    - $\hat{E} = E = 35\% = E[GeV]$ (had)
- $\mu$ chambers
- and much more...

Both detectors asymmetric
Kinematic Regions of DIS
NC and CC incl. Processes measured at HERA

NC: \[ e^\pm + p \rightarrow e^\pm + X, \quad CC: \ e^\pm + p \rightarrow \bar{\nu}_e (\nu_e) + X \]
Measurement of $F_2^\gamma(x,Q^2)$

- For $Q^2 \ll M_Z^2 \rightarrow xF_3$ negligible;
- $F_L$ only important at high $y$;
- Both $F_L$ and $xF_3 \sim$ calculable in QCD
- Correct for higher order QED radiation
- Extract $F_2(x,Q^2)$ from measurement of $\frac{d^2\sigma^{ep}}{dx dQ^2}$

These are difficult measurements: nevertheless precision level has reached: errors of 2-3%
A major finding at Hera: rise of $F_2(x,Q)$ at small $x$

Early fixed-target results:

$Q^2 = 15\, \text{GeV}^2$

$F_2$ rise towards low-$x$ established with $\sim 20\, \text{nb}^{-1}$ in early Hera run

Recent precise determination of $F_2$ (1996-97 data samples)
At high-$Q^2$ still statistics limited... → priority to the measurements at high-$Q^2$
Physical Interpretations of DIS
Structure Function measurements

- The Parton Model (Feynman-Bjorken)
- Theoretical basis of the parton picture and the QCD improved parton model

high energy (Bjorken) limit
Parton Model results on Structure Functions

\[ F_\lambda(x, Q^2) \sim \int_0^1 \frac{d\xi}{\xi} \sum_a f_A^a(\xi) \tilde{F}_\lambda^a(x/\xi, Q^2) + \mathcal{O}\left(\frac{m}{Q}\right). \]

where \( \tilde{F}_\lambda^a(z, Q^2) \) is the “partonic structure function” for DIS on the parton target \( a \).

The Feynman diagram contributing to this elementary quantity and the result of a straightforward calculation are (for electro-magnetic coupling case):

\[ \begin{align*}
\tilde{F}_T^a(x/\xi, Q^2) &= Q_a^2 \delta(x/\xi \leftrightarrow 1) \\
\tilde{F}_L^a(x/\xi, Q^2) &= 0 \\
\tilde{F}_{PV}^a(x/\xi, Q^2) &= 0
\end{align*} \]

⇒ the simple scaling parton model results:

\[ \begin{align*}
F_T(x, Q^2) &= \sum_a Q_a^2 f_A^a(x) \quad \text{(Bj. } \leftrightarrow \text{Feynman)} \\
F_L(x, Q^2) &= 0 \quad \text{(Callan } \leftrightarrow \text{Gross)}
\end{align*} \]
Features of the DIS structure functions due to SM couplings

For CC processes, only one helicity vertex (out of $2 \times 2 \times 3 = 12$) is non-zero:

$$j_L^L(Q) = j_{-1/2}^{-1}(Q) = \sqrt{8Q^2}$$

In Brick-Wall (Breit) frame:

Lorentz boost with $v = \tanh \psi$

Allowed spin configurations:

$$
\begin{align*}
\mu^- & \quad W^+ \\
\nu & \\
u & \\
\bar{u} & \quad W^+ \\
d & \quad W^+ \\
\end{align*}
$$

Vector $\gamma$ coupling:

<table>
<thead>
<tr>
<th>$g_V$</th>
<th>$Q_i T_{3L}^i - 2Q_i \sin^2 \theta_W$</th>
<th>$1 \cdot V_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$</td>
<td>$0$</td>
<td>$T_{3L}^i$</td>
</tr>
<tr>
<td>$g_R$</td>
<td>$\frac{Q_i}{2}$ $- Q_i \sin^2 \theta_W$</td>
<td>$0$</td>
</tr>
<tr>
<td>$g_L$</td>
<td>$\frac{Q_i}{2}$ $T_{3L}^i - Q_i \sin^2 \theta_W$</td>
<td>$1 \cdot V_{ij}$</td>
</tr>
</tbody>
</table>
Consequences on CC Cross sections (parton model level):

\[ \frac{d\sigma^{\nu}/d}{dx dy} \propto W \cdot L \propto F_{\nu/\bar{\nu}} \left( \frac{1 \pm \cosh \psi}{2} \right)^2 \]

\[ \cosh \psi = \frac{2 - y}{y} \quad \Rightarrow \quad \frac{1 \pm \cosh \psi}{2} \propto \left\{ \begin{array}{c} 1 \\ 1 - y \end{array} \right\} \]

\[ \frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = \int dy \frac{d\sigma^{\bar{\nu}}}{dy} / \int dy \frac{d\sigma^{\nu}}{dy} \approx \frac{1}{3} \]

These qualitative features were verified in early (bubble chamber) high energy neutrino scattering experiments.

**Gargamelle** (CERN)

Refined measurements reveal QCD corrections to the approximate naïve parton model results. These are embodies in all modern “QCD fits” and “global analyses”. 
Features of the partonic interactions revealed by DIS experiments have firmly established that the lepton probes interact primarily with spin ½ quark partons inside the nucleons with couplings of the SM.

Leading (diagonal) EM NC scattering processes:

\[ \frac{d\sigma}{dx dy} \propto F_T (1 + \cosh^2 \psi) \]

(y is the hyperbolic “angle” connecting the lepton and hadron vertices.)

Analogous to the familiar angular distribution of scattering of spin ½ elementary particles in the CM frame:

\[ \frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) \]

At HERA, the \( \gamma-Z \) interference term also contribute, giving rise to more complicated patterns for the “angular” (y) distribution.

Features of the partonic interactions revealed by DIS experiments have firmly established that the lepton probes interact primarily with spin ½ quark partons inside the nucleons with couplings of the SM.
Structure functions: Quark Parton Model

Quark parton model (QPM) NC SFs for proton target:

\[
[F_2^\gamma, F_2^{\gamma Z}, F_2^Z] = \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2] \{q + \bar{q}\}
\]

\[
[x F_3^{\gamma Z}, x F_3^Z] = 2x \sum_q [e_q a_q, v_q a_q] \{q - \bar{q}\} = 2x \sum_{q=u,d} [e_q a_q, v_q a_q] q_v
\]

QPM CC SFs for proton targets:

\[
xF_{2,W+}^{CC} = x\{\bar{u} + \bar{c} + d + s\}, \quad xF_{3,W+}^{CC} = x\{d + s - (\bar{u} + \bar{c})\}
\]

\[
xF_{2,W-}^{CC} = x\{u + c + (\bar{d} + \bar{s})\}, \quad xF_{3,W-}^{CC} = x\{u + c - (\bar{d} + \bar{s})\}
\]

For neutron targets, invoke (flavor) isospin symmetry:

\[
u \Leftrightarrow d \quad \text{and} \quad \bar{u} \Leftrightarrow \bar{d}
\]
High-Q² CC cross section from HERA

\[ e^+ + p \rightarrow \bar{\nu} + X \]

\[ e^- + p \rightarrow \nu + X \]

**Charged Current**

- H1 e⁺p 99-00 \( \sqrt{s} = 319 \text{ GeV} \)
- H1 e⁺p 94-97 \( \sqrt{s} = 301 \text{ GeV} \)

**ZEUS**

(a)

- ZEUS 99-00 e⁺p \( \rightarrow \bar{\nu}_e X \)

(b)

- ZEUS-S
Comparing NC and CC Xsec’s at HERA: EW Unification

NC cross section sharply decreases with decreasing $Q^2$ (dominant $\gamma$ exchange):

$$\sim \frac{1}{Q^4}$$

CC cross section approaches a constant at low $Q^2$

$$\sim \left[ \frac{M_W^2}{(Q^2 + M_W^2)} \right]^2$$

Dramatic confirmation of the unification of the electromagnetic and weak interactions of the SM in Deep Inelastic Scattering.
Manifestation of $\gamma Z$ interference: $x F_3^N C$ (NC) at Hera

\[ x F_3^{NC} = \frac{1}{2 Y_-} [\tilde{\sigma}(e^- p) - \tilde{\sigma}(e^+ p)] \]

\[ \frac{d^2 \sigma_{\text{Born}}^{NC}(e^\pm p)}{dx dQ^2} = \frac{2\pi \alpha^2}{x Q^4} [Y^+ F_2^{NC}(x,Q^2) - y^2 F_L^{NC}(x,Q^2) \mp Y x F_3^{NC}(x,Q^2)] \]

Needs better $e^-$ data!
\[
\begin{align*}
\gamma Z (NC) \\
\text{Data H1 PDF 2000} \\
\bullet & \quad Q^2 = 1500 \text{ GeV}^2 \\
\quad & \quad 5000 \text{ GeV}^2 \\
\quad & \quad 12000 \text{ GeV}^2 \\
\text{transformed to } Q^2 = 1500 \text{ GeV}^2 \\
\bullet & \quad H1 \\
\quad & \quad H1 \text{ PDF 2000}
\end{align*}
\]

\[\begin{align*}
\gamma Z \\
\begin{align*}
\gamma Z \\
\gamma Z \\
\end{align*}
\end{align*}
\]

\[
x F_3^{\gamma Z} (NC) = -a_e \frac{\kappa Q^2}{(Q^2 + M_Z^2)} x F_3^{\gamma Z} + (2\nu_e a_e) \left( \frac{\kappa Q^2}{Q^2 + M_Z^2} \right)^2 x F_3^Z
\]

\[
x F_3^{\gamma Z} \approx -F_3^{\gamma Z} (Q^2 + M_Z^2) / (a_e \kappa Q^2)
\]
Helicity structure

\[ e^-q \quad e^+q \]
\[ \Rightarrow L.H. + L.H. \quad R.H. + L.H. \]
\[ \Rightarrow \text{in CMS} \quad \sum_{e^-q} S_i = 0 \quad = 1 \]
\[ \Rightarrow \text{isotropic in } \theta_{CMS} \quad \text{peaked forward} \]

Down-type (anti-)quarks contribution suppressed by helicity:
\[ \tilde{\sigma}^{e^-p} = x \left[ u + c + (1-y)^2(\bar{d} + s) \right] \]
\[ \tilde{\sigma}^{e^+p} = x \left[ \bar{u} + \bar{c} + (1-y)^2(d + s) \right] \]

Helicity-structure of EW confirmed

Assuming \( q_s = \bar{q}_s \Rightarrow \)
\[ \tilde{\sigma}^{e^-p} - \tilde{\sigma}^{e^+p} = xu_v - (1 - y)^2 x d_v \]
\[ \Rightarrow \text{access to valence PDFs} \]

Needs e\(^-\) data
QCD and DIS

Cf. Introductory course by Sterman

Master Equation for QCD Parton Model – the Factorization Theorem

\[ F_A^\lambda(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes \hat{F}_a^\lambda(x, \frac{Q}{\mu}, \frac{M}{Q}) + O\left(\frac{\Lambda}{Q}\right)^2 \]

\( \mu \) is the factorization scale.

Usually choose \( \mu = Q \): that is how \( f(x, Q) \) acquires \( Q \)-dep.

Experimental Input

Hard Cross-section perturbative, calculable (may contain \( \alpha_s^n \log^n(M/Q) \))

universal Parton Dist. Fn.

Non-Perturbative Parametrization at \( Q_0 \)

GLAP Evolution to \( Q \)

extracted by global analysis
$F_2$ : “Scaling violation”  
— Q-dependence inherent in QCD

Renormalization group equation governs the scale dependence of parton distributions and hard cross sections.  
(DGLAP)

- Rise with increasing $Q$ at small-$x$
- Flat behavior at medium $x$
- Decrease with increasing $Q$ at high $x$
QCD evolution

Evolution performed in terms of (1/2/3) non-singlet, singlet and gluon densities:

\[
\frac{\partial}{\partial \ln \mu_F^2} q^{\pm}_{NS} = P_{NS}^{\pm} \otimes q_{NS}^{\pm}
\]

\[
\frac{\partial}{\partial \ln \mu_F^2} \left\{ \sum g \right\} = \left\{ \begin{array}{c} P_{qg} \\ P_{gg} \end{array} \right\} \otimes \left\{ \sum g \right\} = P \otimes q
\]

Where

\[
P(x) = a_s P^{(0)}(x) + a_s^2 \left[ P^{(1)}(x) - \beta_0 \ln \frac{\mu_F^2}{\mu_R^2} P^{(0)}(x) \right]
\]

\[
\frac{da_s}{d \ln \mu_R^2} = \beta(a_s) = \sum_{i=0}^{\infty} a_s^{i+2} \beta_i \approx a_s^2 \beta_0 + a_s^3 \beta_1
\]

with \( a_s = \frac{\alpha_s(\mu_R^2)}{4\pi} \)

where \( \beta_0 = 11 - \frac{2}{3} N_F \)

and \( \beta_1 = 102 - \frac{38}{3} N_F \)

Cf. Introductory course by Sterman
Parton Distribution Functions (PDF): most significant physical results derived from DIS (with help from other hard scattering processes)

A common misconception:
Parton distribution functions ≠ “Structure functions”

These are the (process-dep) S.F.s

These are the (universal) PDFs

These are the hard Xsecs.

There is a convolution integral and a summation over partons here!
Overview of Parton Distribution Functions of the Proton

CTEQ5M

Q = 5 GeV

\( x f(x,Q) \)

\( 10^{-4} \) to \( 10^{-1} \)

\( 0 \) to \( 1.2 \)

Gluon / 15

d_{bar}

u_{bar}

s

c

u_v

d_v

(d_{bar} - u_{bar}) * 5

Cf. course on global analysis and PDFs
Outline of the course:

- **Basic Formalism**
  - (indep. of strong dynamics and parton picture)

- **Experimental Development**
  - Fixed target experiments
  - HERA experiments

- **Parton Model and QCD**
  - Parton Picture of Feynman-Bjorken
  - Asymptotic freedom, factorization

- **Phenomenology**
  - QCD parameters
  - Parton distribution functions
  - Other interesting topics

---

**Summary and Conclusion**

Important to know the model indep. foundation of the measured structure functions and their basic properties.

There is a long and distinguished history, dating back to Rutherford.

These highest energy and highest statistics expts. provide the basis for modern precision phenomenology.

DIS experiments provided direct evidence for the parton structure of the nucleon, and confirmed every aspect of the SU(3)xSU(2)xU(1) SM.

Cf. the rest of the Summer School courses for exciting consequences of PQCD as well as other modern theories to follow.